Conjunctive Query Processing
[A Formal Model for Theoretical Focus]

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Motivation for the Model

- Given a query on k relations, each of n rows (i.e., a k-way join), naively
  - Processing time: $O(n^k)$
  - Size of the output, also, $O(n^k)$
- If basic complexity models are our guide, even simple queries should be infeasible (e.g. n = 1,000,000 and k = 5)

What happens in practice?

- Joins are often with high reduction factor (i.e., low selectivity)
- Example: $R \bowtie S$ on the the primary key p of R
  - Assume the selectivity for p is $\frac{1}{n}$ (i.e., there is 1 output result for each primary key of R)
  - Output size estimation is no longer $O(n^2)$ but $O(n)$ ($\frac{1}{n} \times n^2$)
- Relational queries usually work subject to good optimization choices
  - → can still be slow
  - → can be volatile in their performance

Conjunctive Queries (CQ)

- A subset of relational algebra
- Goals of studying CQ
  - Enable theoretical study of the algorithmically hard part of queries
  - Help explain (and thus help resolve) peculiar system behavior
  - Develop new algorithms and *hopefully* impact practice

Full Conjunctive Query

- In Relational Algebra
  - Natural join of l relations with $O(n)$ tuples each, no projection
  - $Q(A_1, A_2, A_3, A_4) = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$
- In Datalog
  - $Q(A_1, A_2, A_3, A_4) \leftarrow R_1(A_1, A_2), R_2(A_1, A_2, A_3), R_3(A_2), R_4(A_1, A_2, A_4)$
- In SQL, full CQ = `SELECT` … `FROM` … `WHERE` statement
  - `WHERE` contains only equalities
  - No projection

Full Conjunctive Query

Other parameters:

- Query size: $O(l)$ (e.g., $l = 4$ for above query)
- Join output result size cardinality: r

With Tight Focus on the Computational Challenge

- Main concern: come up algorithms that can evaluate query fast
- Query evaluation problem is known to be NP-Complete
  - No algorithm exists to evaluate <u>any possible query</u> correctly and runs in polynomial time
  - Not a death sentence yet!
  - NP-Complete → algorithm cannot have <u>all</u> three properties
    - *General purpose.* The algorithm accommodates all possible inputs of the computational problem
    - *Correct.* For every input, the algorithm correctly solves the problem.
    - *Fast.* For every input, the algorithm runs in polynomial time.
- Choose one to compromise – General Purpose

A Critical Special Case: Acyclic Conjunctive Query

- CQs into fall two classes
  - Acyclic CQ
  - Cyclic CQ
- A **polynomial** algorithm exists to evaluate acyclic CQ
  - Yannakakis Algorithm – a three-pass algorithm
    - $O(\max(r, kn))$ where r is the size of the output, kn is the size of the input

Acyclicity

- A query is acyclic iff it has at least one of these properties
    1. a join tree
    2. a full reducer
    3. An acyclic hypergraph*

* Historically, query acyclicity was independently defined with different notations. They are shown to be equivalent.

Running example

$$Q(A_1, A_2, A_3, A_4) \leftarrow R_1(A_1, A_2), R_2(A_1, A_2, A_3), R_3(A_2), R_4(A_1, A_2, A_4)$$

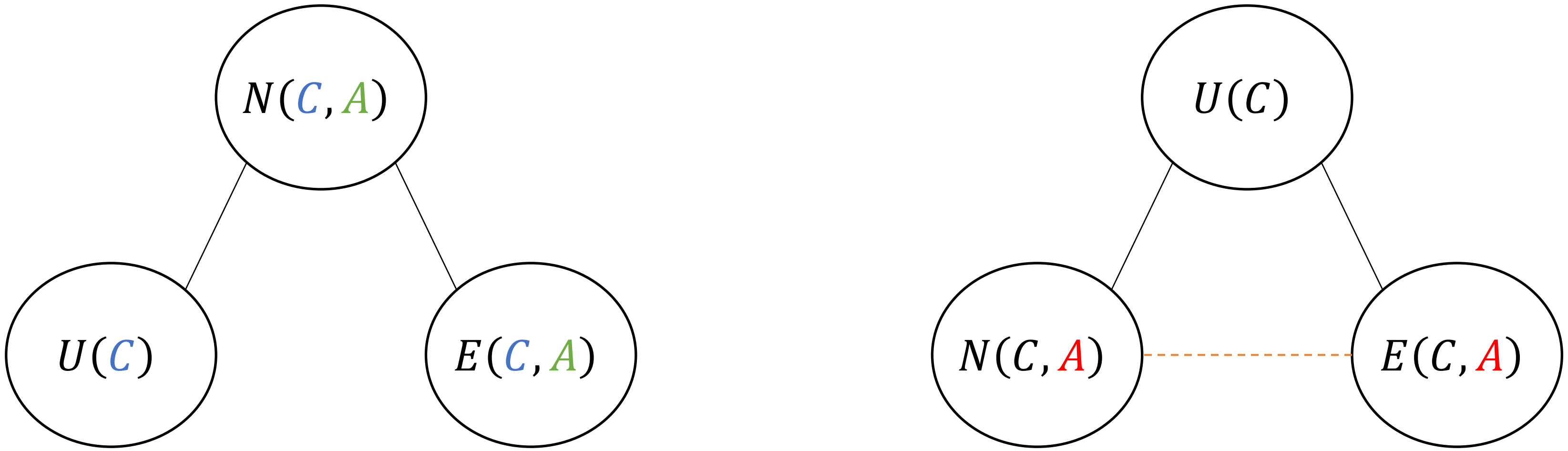
- Goal: show Q is acyclic through three properties above

Property 1: query has a join tree

- Join tree = acyclic query graph + *connectedness condition*
- query graph – introduced and leveraged for DP-based query opt.
  - Relations are nodes
  - Edges are joins
- *Connectedness condition:*
  - Def 1: For each attribute A, the nodes containing A form a connected subtree
  - Def 2: For each pair of nodes R and S that have common attributes, the following conditions hold:
    - R and S are connected
    - All variables common to R and S occur on the unique path from R to S

Example

- Suppose we have a database that contains $U(C), N(C, A), E(C, A)$



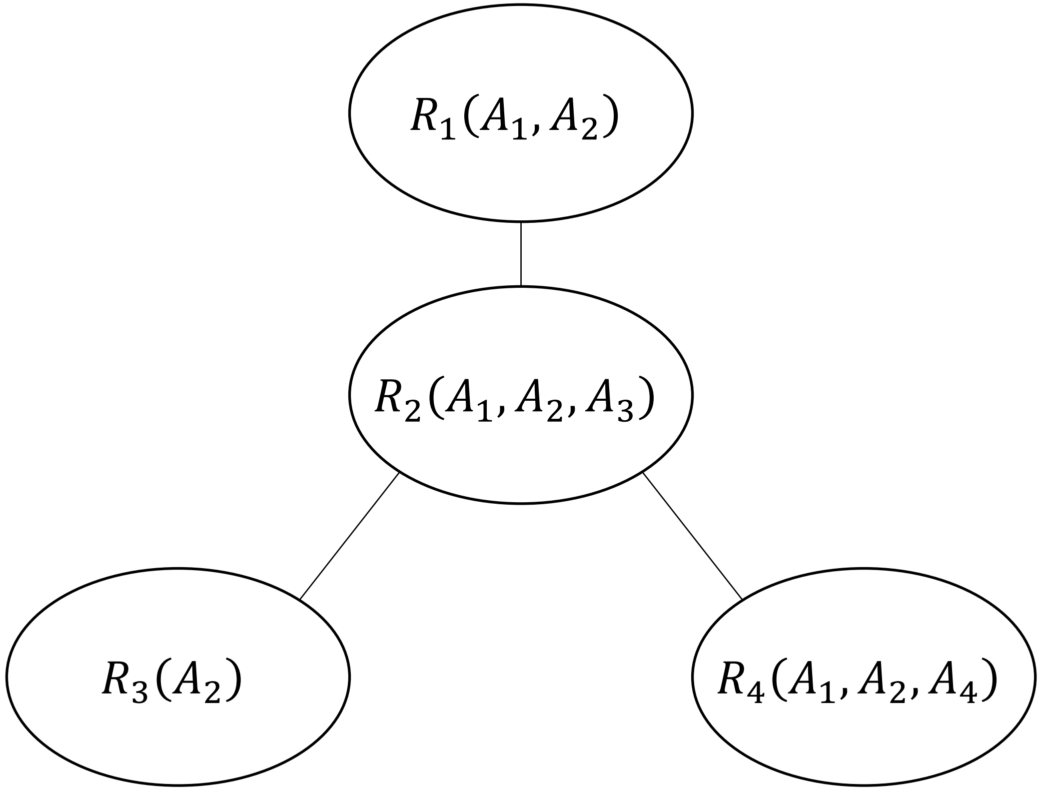
Join tree

Not a Join tree

- A query is acyclic if we can find a join tree
  - can be done in linear time!

Example

- $Q(A_1, A_2, A_3, A_4) \leftarrow R_1(A_1, A_2), R_2(A_1, A_2, A_3), R_3(A_2), R_4(A_1, A_2, A_4)$ is acyclic because we can find a join tree

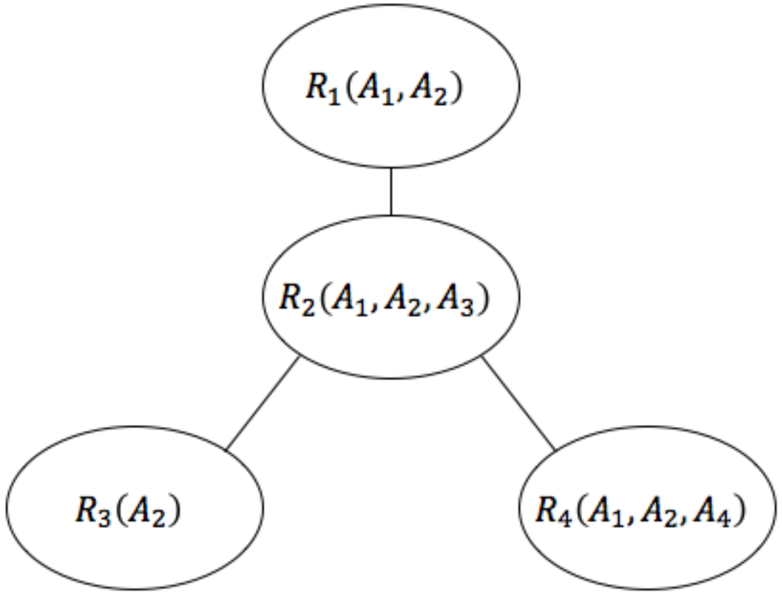


Property 2: query has a full reducer

- A full reducer = a semi-join program that remove all dangling tuples in relations
  - Semi-join program = a set of semi-join operations (i.e., semi-join reduction)
  - Dangling tuples = tuples that are not part of final join result
- Example:
  - $Q(A_1, A_2, A_3, A_4) \leftarrow R_1(A_1, A_2), R_2(A_1, A_2, A_3), R_3(A_2), R_4(A_1, A_2, A_4)$ has a full reducer (and thus acyclic)
    - $R_2 \ltimes R_4, R_2 \ltimes R_3, R_1 \ltimes R_2, R_2 \ltimes R_1, R_3 \ltimes R_2, R_4 \ltimes R_2$
    - Full reducer doesn't depend on the actual data of each relation!
    - How do you find a full reducer?

Find a full reducer – a two pass process

- $Q(A_1, A_2, A_3, A_4) \leftarrow R_1(A_1, A_2), R_2(A_1, A_2, A_3), R_3(A_2), R_4(A_1, A_2, A_4)$
- Suppose we have a join tree of Q, we can construct a full reducer by
  - Semi-join reduction sweep from leaves to root
    - $R_2 \ltimes R_4, R_2 \ltimes R_3, R_1 \ltimes R_2$
  - Semi-join reduction sweep from root to leaves
    - $R_2 \ltimes R_1, R_3 \ltimes R_2, R_4 \ltimes R_2$
- Will this work?



Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

R_1

A_1	A_2
1	20
1	10
4	60

A_1, A_2

R_2

A_1	A_2	A_3
1	10	100
1	20	100
3	10	300
1	40	300
2	30	200

A_2

A_1, A_2

R_3

A_2
10
20
30

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
2	20	2000

Slides of this example are from DATA Lab@Northeastern University

Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

1. Bottom-up traversal (semi-joins)

R_1

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1	20
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A_1, A_2

R_2

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1	10	100
1	20	100
3	10	300
1	40	300
2	30	200

$R_2 \ltimes R_4$

A_2

R_3

A_2
10
20
30

A_1, A_2

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
2	20	2000

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Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

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2	30	200

A_2

R_3

A_2
10
20
30

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
2	20	2000

$\boldsymbol{R_2 \ltimes R_4}$

A_1	A_2
1	10
1	20
2	20

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Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

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A_1, A_2

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1	10	100
1	20	100
~~3~~	~~10~~	~~300~~
~~1~~	~~40~~	~~300~~
~~2~~	~~30~~	~~200~~

A_2

R_3

A_2
10
20
30

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
2	20	2000

$R_2 \ltimes R_4$

A_1	A_2
1	10
1	20
2	20

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Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

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~~1~~	~~40~~	~~300~~
~~2~~	~~30~~	~~200~~

$R_2 \ltimes R_3$

A_2
10
20
30

$R_2 \ltimes R_4$

A_1	A_2
1	10
1	20
2	20

R_3

A_2
10
20
30

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
2	20	2000

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Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

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A_1	A_2
1	20
1	10
~~4~~	~~60~~

$R_1 \ltimes R_2$

A_1	A_2
1	10
1	20

R_2

A_1	A_2	A_3
1	10	100
1	20	100
~~3~~	~~10~~	~~300~~
~~1~~	~~40~~	~~300~~
~~2~~	~~30~~	~~200~~

$R_2 \ltimes R_3$

A_2
10
20
30

$R_2 \ltimes R_4$

A_1	A_2
1	10
1	20
2	20

R_3

A_2
10
20
30

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
2	20	2000

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Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

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1	20
1	10

$R_1 \ltimes R_2$

A_1	A_2
1	10
1	20

R_2

A_1	A_2	A_3
1	10	100
1	20	100

$R_2 \ltimes R_3$

A_2
10
20
30

$R_2 \ltimes R_4$

A_1	A_2
1	10
1	20
2	20

R_3

A_2
10
20
30

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
2	20	2000

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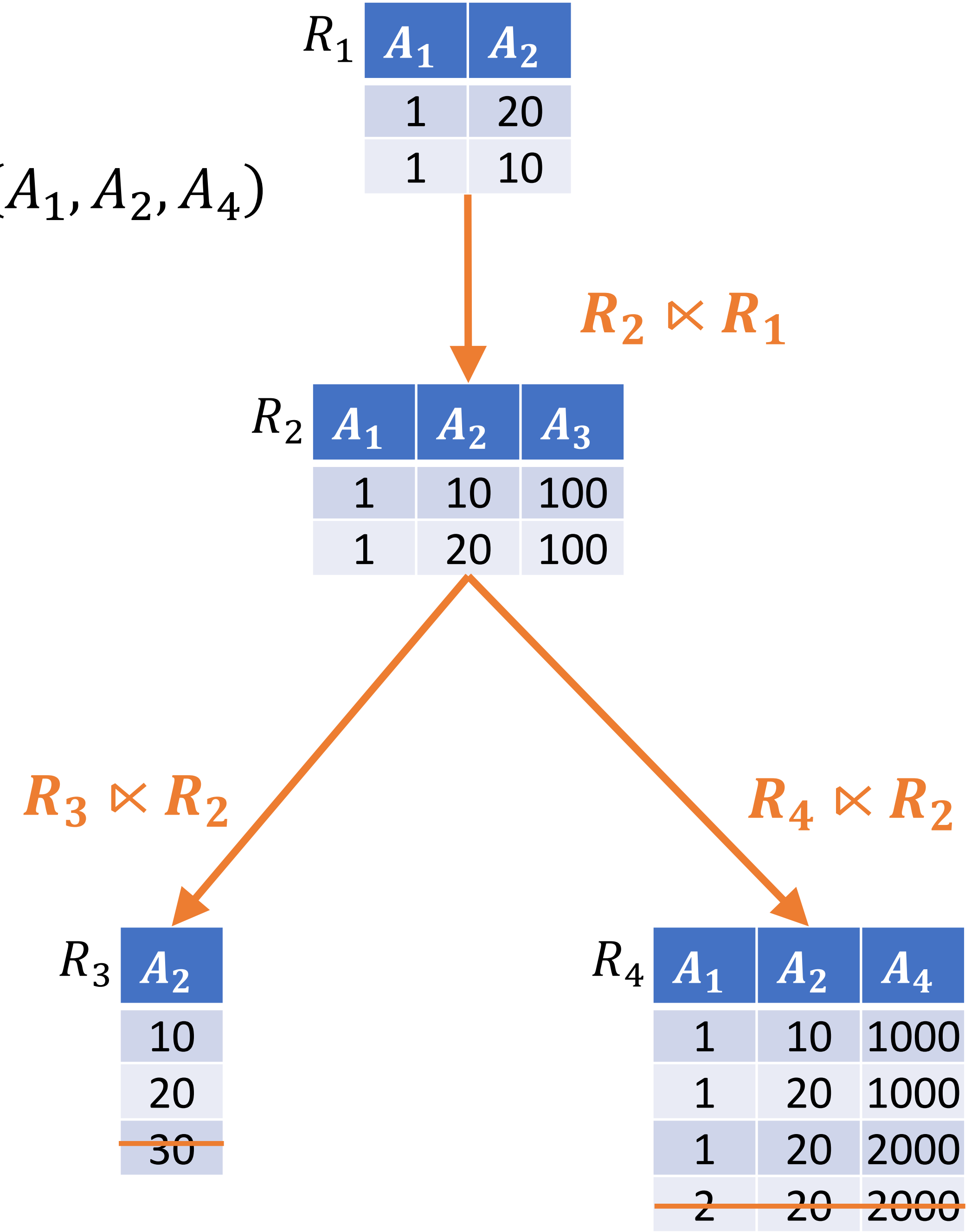
Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

1. Bottom-up traversal (semi-joins)
2. Top-down traversal (semi-joins)



R_1

A_1	A_2
1	20
1	10

R_2

A_1	A_2	A_3
1	10	100
1	20	100

R_3

A_2
10
20
~~30~~

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000
~~2~~	~~20~~	~~2000~~

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Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

1. Bottom-up traversal (semi-joins)
2. Top-down traversal (semi-joins)

R_1

A_1	A_2
1	20
1	10

R_2

A_1	A_2	A_3
1	10	100
1	20	100

R_3

A_2
10
20

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000

Slides of this example are from DATA Lab@Northeastern University

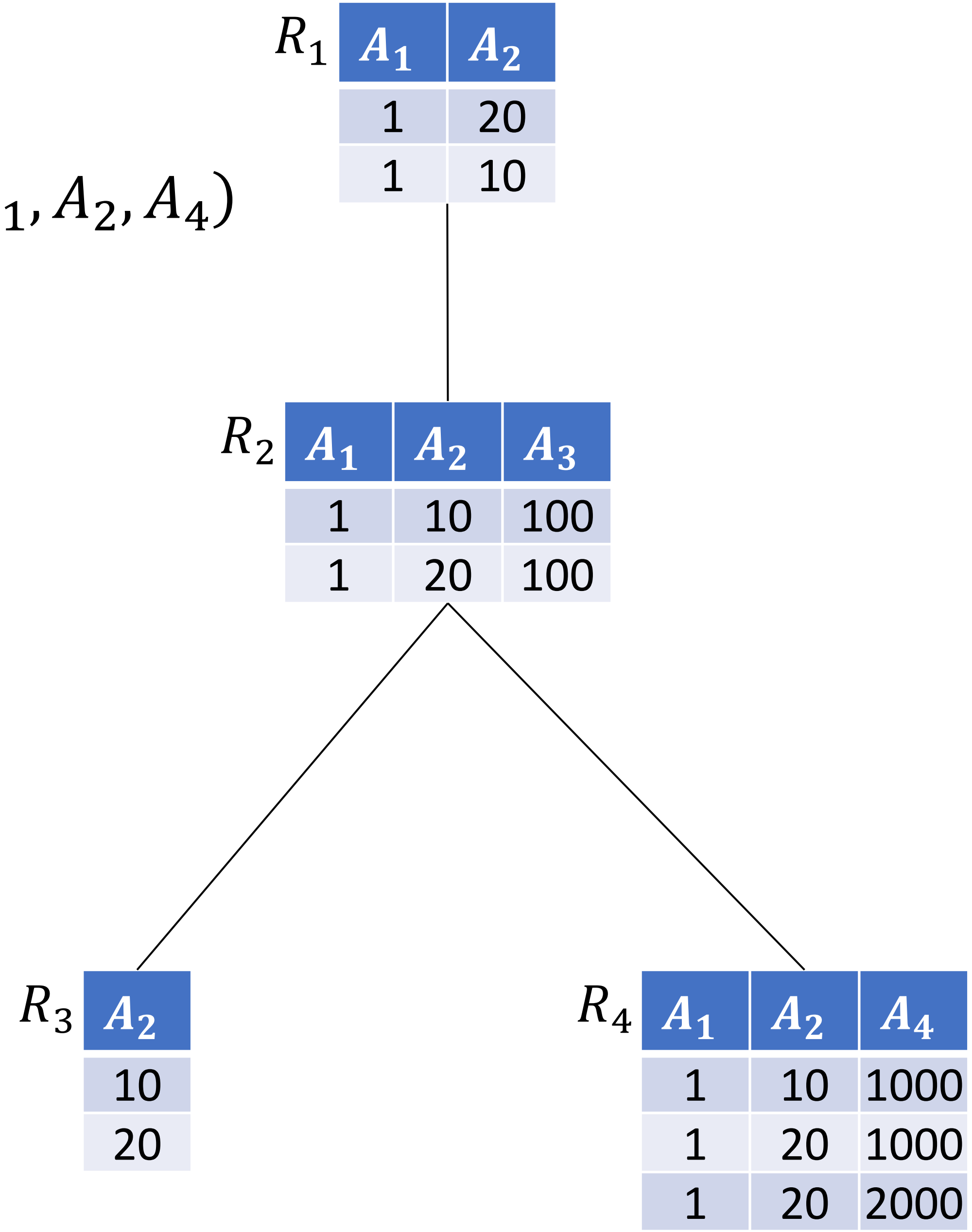
Yannakakis Algorithm

- Given acyclic conjunctive query represented by a join tree
- Two Phases
  - Apply a full reducer based on join tree
    - Semi-join reduction sweep from leaves to root
    - Semi-join reduction sweep from root to leaves
  - Use the join tree as the query plan and compute the joins bottom up

Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

1. Bottom-up traversal (semi-joins)
2. Top-down traversal (semi-joins)



R_1

A_1	A_2
1	20
1	10

R_2

A_1	A_2	A_3
1	10	100
1	20	100

R_3

A_2
10
20

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000

Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

1. Bottom-up traversal (semi-joins)
2. Top-down traversal (semi-joins)
3. Join bottom-up

$$R_2 = R_3 \bowtie R_2$$

$$R_2 = R_4 \bowtie R_2$$

$$R_1 = R_1 \bowtie R_2$$

R_1

A_1	A_2
1	20
1	10

R_2

A_1	A_2	A_3
1	10	100
1	20	100

R_3

A_2
10
20

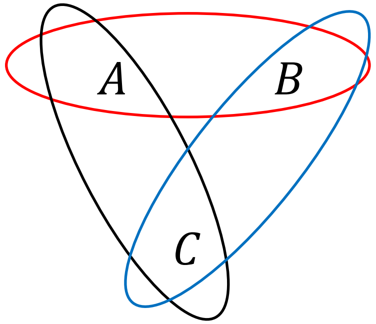
R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000

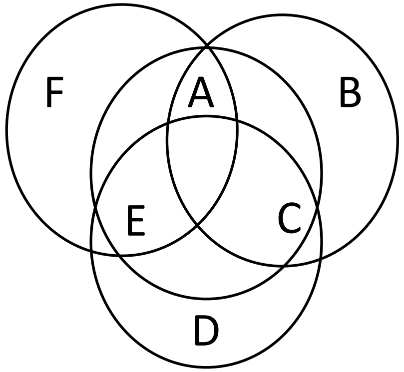
Slides of this example are from DATA Lab@Northeastern University

Property 3: query has an acyclic hypergraph

- A hypergraph for a natural join
  - Node = attribute in query
  - Hyperedge = relation
- Example 1: Triangle Query
  - $Q(A,B,C) \leftarrow R(A,B), S(B,C), T(C,A)$
  - Relation $R(A,B)$ is represented by the hyperedge $\{A,B\}$
  - Relation $S(B,C)$ is represented by the hyperedge $\{B,C\}$
  - This hypergraph is actually a graph, since the hyperedges are each pairs of nodes



- Example2
  - $Q(A,B,C,D,E,F) \leftarrow R(A,E,F), S(A,B,C), T(C,D,E), U(A,C,E)$



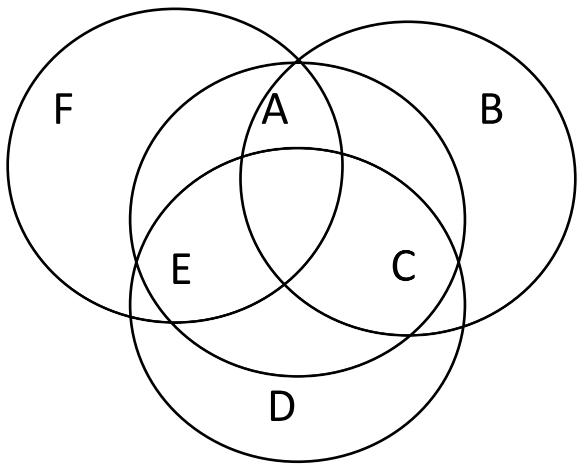
Hypergraph construction a legacy of “The Universal Relation” war.

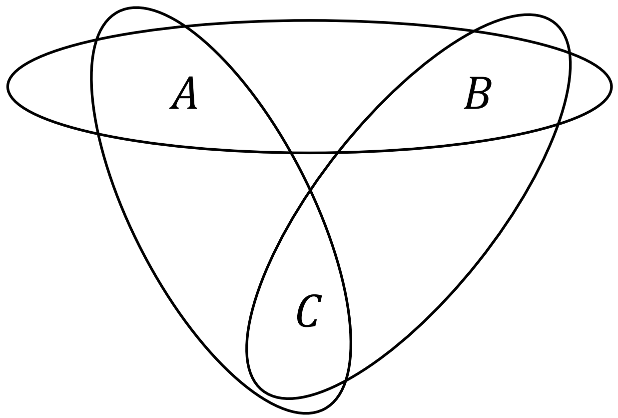
- Universal Relation: A concept where all relation schema would be removed and all data merged into a single table.
  - Plausibility: compute cross products as needed, and fill in implausible combinations with NULLs
  - Potential benefit: Obtain certain optimal properties that might not be achievable without removing certain input from a developer.

Hypergraph definition (cont')

- To define acyclic hypergraph, we need the notion of an "ear" in a hypergraph
- A hyperedge H is an *ear* if there is some other hyperedge G in the same hypergraph such that every node of H is either:
  - Found only in H, or
  - Also found in G
- We shall say that G *consumes* H

Ear in Hypergraph Examples





Hyperedge $H = \{A, E, F\}$ is an ear

- $G = \{\mathrm{A, C, E}\}$
- Node F is unique to H; it appears in no other hyperedge
- The other two nodes of H (A and E) are also members of G
- What about $\{A, B, C\}, \{C, D, E\}$?

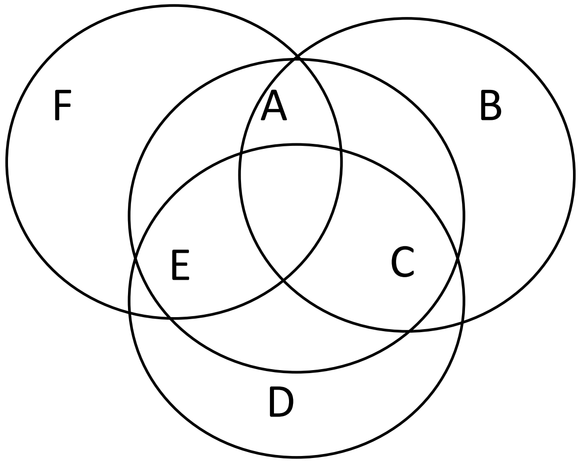
Find ears in this hypergraph

Check Cyclicity of Hypergraph: GYO Algorithm

- GYO Algorithm = a sequence of ear reductions
- An ear reduction = the elimination of one ear from the hypergraph, along with any nodes that appear only in that ear
- A hypergraph is acyclic = the output of GYO algorithm is empty
  - i.e., all hyperedges can be removed by ear reductions
- Properties
  - An ear, if not eliminated at one step, remains an ear after another ear is eliminated
  - Hyperedge that was not an ear, can become an ear after another hyperedge is eliminated

Example

- $\{A, E, F\}$, $\{A, B, C\}$, $\{C, D, E\}$ are ears
- Pick one and eliminate it
- Suppose we pick $\{A, E, F\}$



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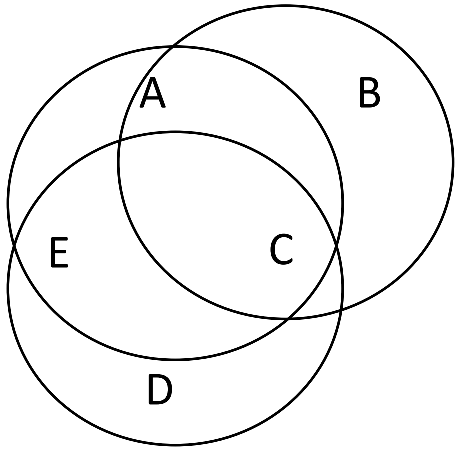
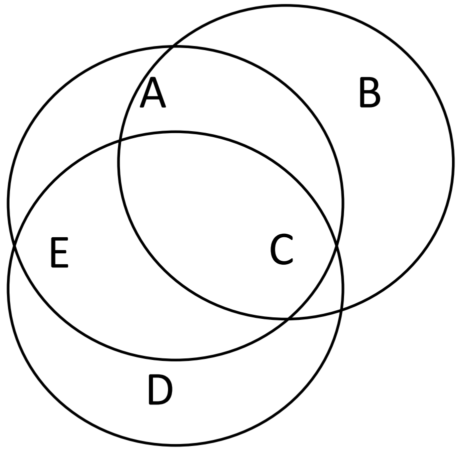


Example

- $\{A, E, F\}$, $\{A, B, C\}$, $\{C, D, E\}$ are ears
- Pick one and eliminate it
- Suppose we pick $\{A, E, F\}$
- Next, we pick $\{A, B, C\}$ and eliminate it



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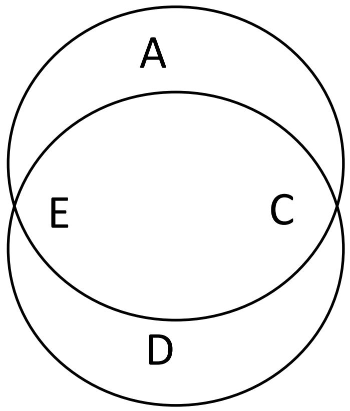
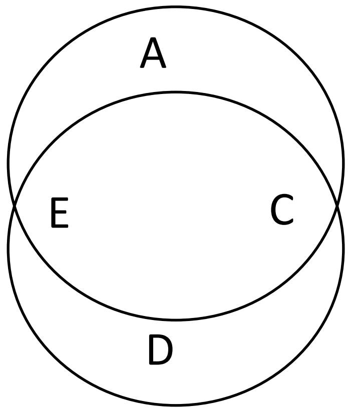


Example

- $\{A, E, F\}$, $\{A, B, C\}$, $\{C, D, E\}$ are ears
- Pick one and eliminate it
- Suppose we pick $\{A, E, F\}$
- Next, we pick $\{A, B, C\}$ and eliminate it
- $\{A, C, E\}$ now becomes an ear and eliminate it



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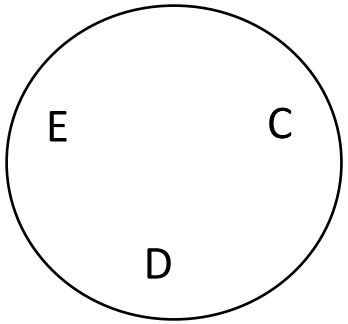

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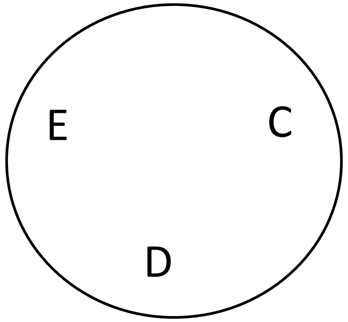
- Pick one and eliminate it
- Suppose we pick $\{A, E, F\}$
- Next, we pick $\{A, B, C\}$ and eliminate it
- $\{A, C, E\}$ now becomes an ear and eliminate it
- $\{C, D, E\}$ is the only left ear and eliminate it



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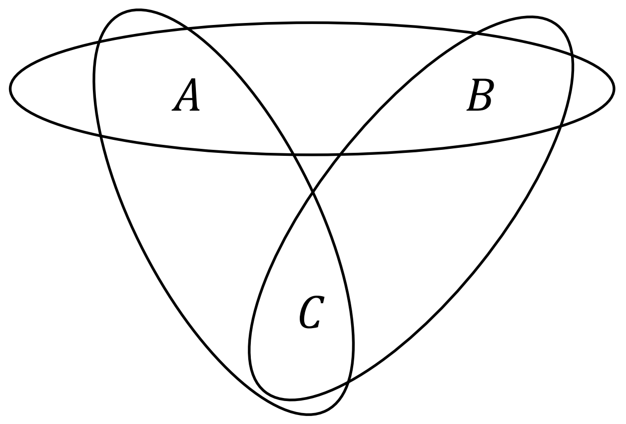
- Next, we pick $\{A, B, C\}$ and eliminate it
- $\{A, C, E\}$ now becomes an ear and eliminate it
- $\{C, D, E\}$ is the only left ear and eliminate it
- Original hypergraph is acyclic

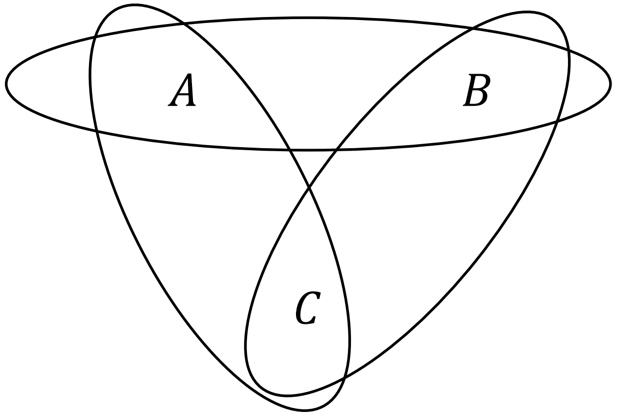
Example 2

- Pick an ear to remove



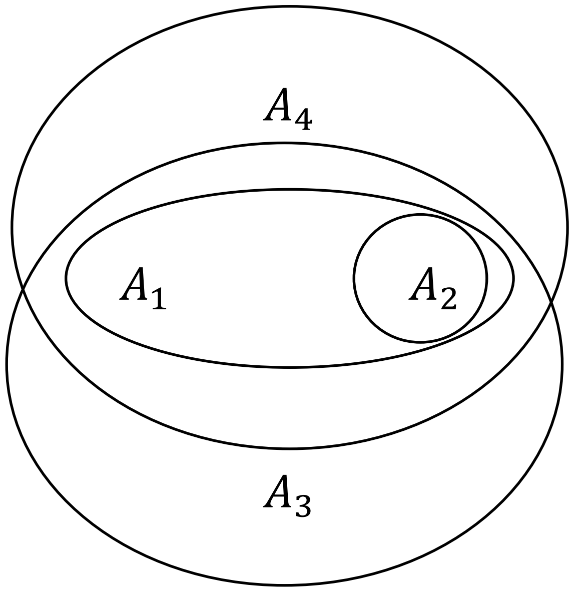
Example 2

- Pick an ear to remove
- No ear to remove → hypergraph is cyclic



Example 3

- $Q(A_1, A_2, A_3, A_4) \leftarrow R_1(A_1, A_2), R_2(A_1, A_2, A_3), R_3(A_2), R_4(A_1, A_2, A_4)$



Sequence of ear reductions
- $\{A_2\}$
- $\{A_1, A_2\}$
- $\{A_1, A_2, A_3\}$
- $\{A_1, A_2, A_4\}$

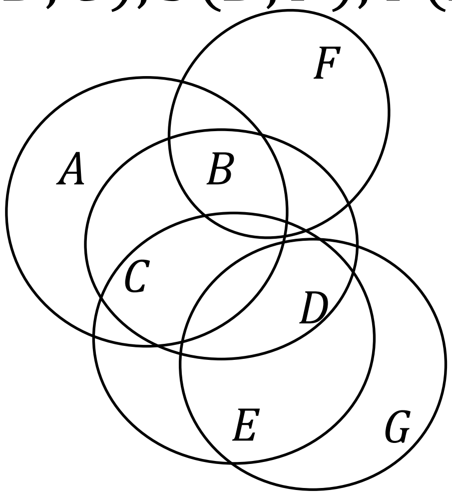
Q is acyclic

Recap

- We have seen three properties for acyclic query
  1. It has a join tree, or
  2. It has a full reducer, or
  3. Its hypergraph is acyclic
- We see how to construct a full reducer from a join tree
- Question: how to find a join tree for a query, if it exists?

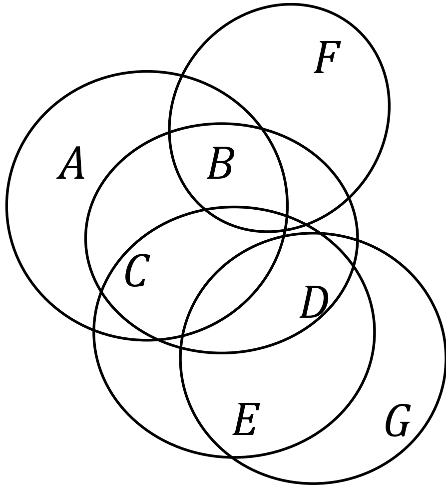
Find a Join Tree

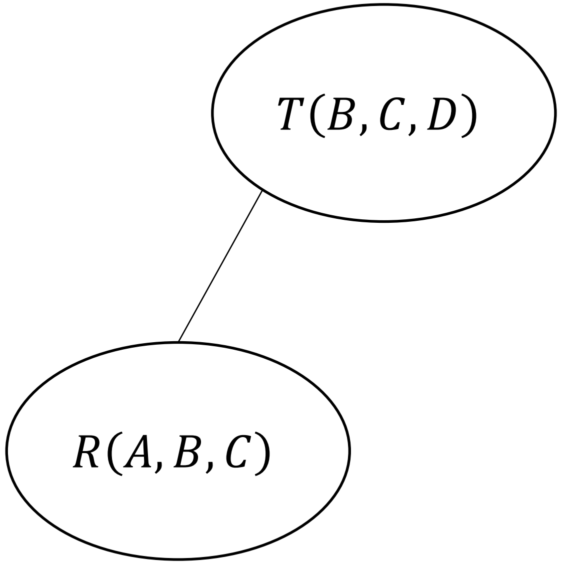
- We can construct a join tree during GYO algorithm. In addition to ear reduction, we follow additional steps:
  - Tree nodes = hyperedges
  - The children of a tree node H are all those hyperedges *consumed* by H
- Example
  - $R(A,B,C), S(B,F), T(B,C,D), G(C,D,E), H(D,E,G)$



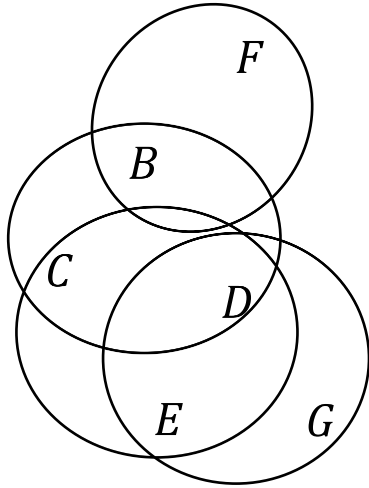
Join Tree 1

- Start to eliminate $\{A, B, C\}$
- Since $\{B, C, D\}$ consumes $\{A, B, C\}$, $\{B, C, D\}$ is the parent of $\{A, B, C\}$

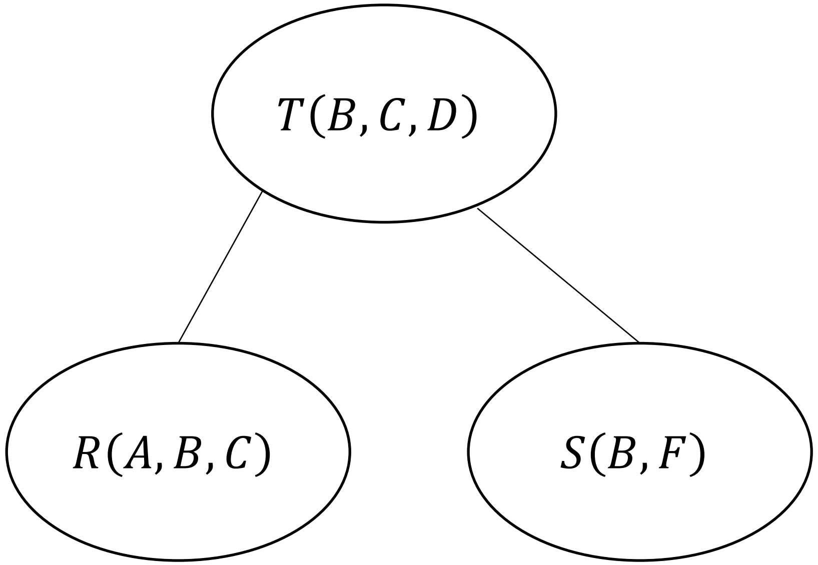




Join Tree 1

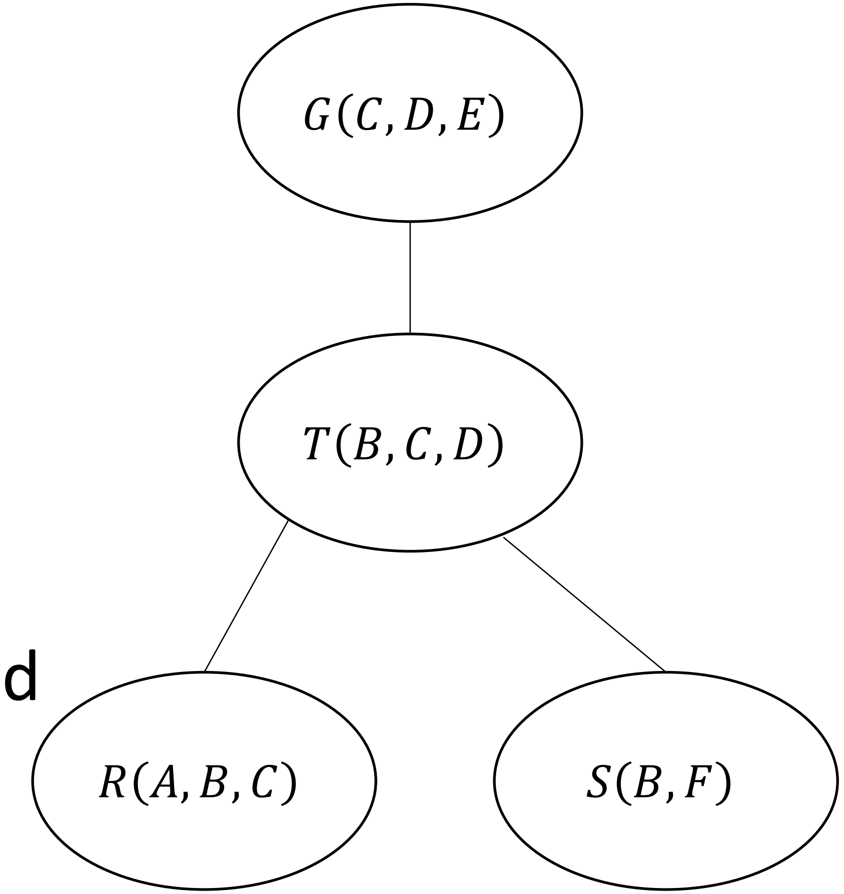


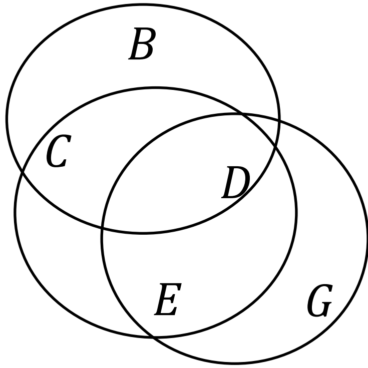
- Start to eliminate $\{A, B, C\}$
- Since $\{B, C, D\}$ consumes $\{A, B, C\}$, $\{B, C, D\}$ is the parent of $\{A, B, C\}$
- Next, remove $\{B, F\}$, which is also consumed by $\{B, C, D\}$



Join Tree 1

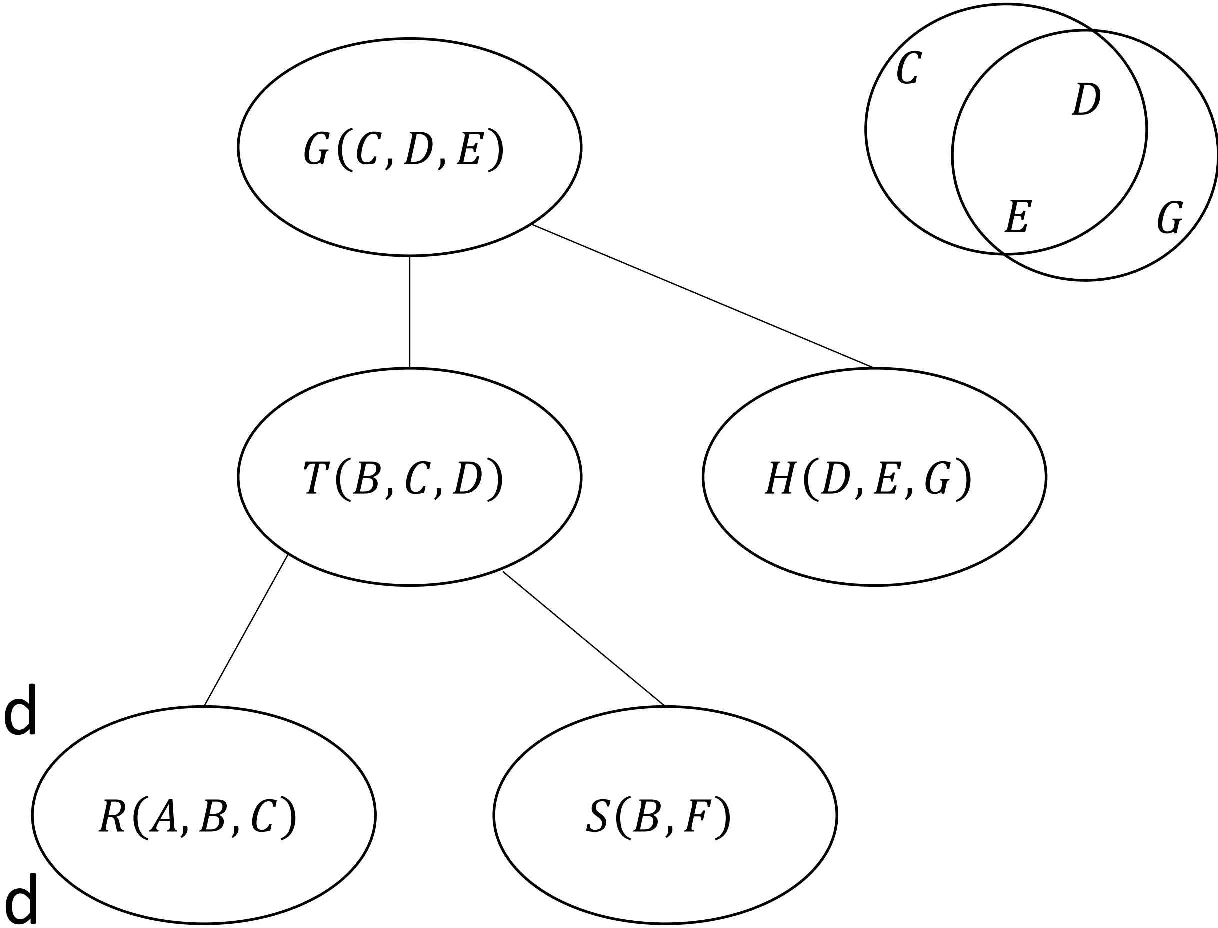
- Start to eliminate $\{A, B, C\}$
- Since $\{B, C, D\}$ consumes $\{A, B, C\}$, $\{B, C, D\}$ is the parent of $\{A, B, C\}$
- Next, remove $\{B, F\}$, which is also consumed by $\{B, C, D\}$
- Remove $\{B, C, D\}$, which is consumed by $\{C, D, E\}$



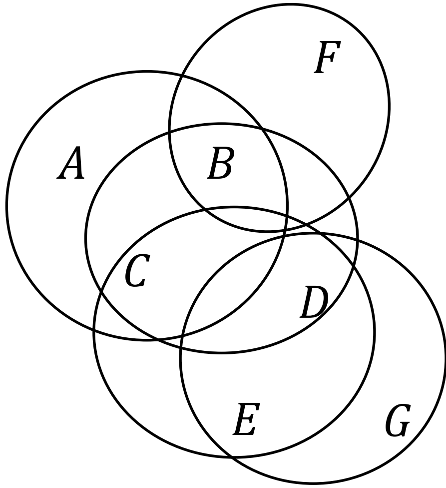


Join Tree 1

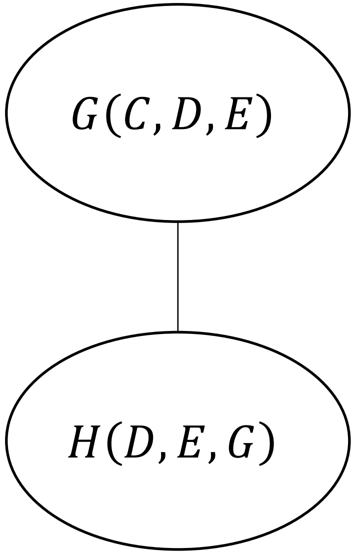
- Start to eliminate $\{A, B, C\}$
- Since $\{B, C, D\}$ consumes $\{A, B, C\}$, $\{B, C, D\}$ is the parent of $\{A, B, C\}$
- Next, remove $\{B, F\}$, which is also consumed by $\{B, C, D\}$
- Remove $\{B, C, D\}$, which is consumed by $\{C, D, E\}$
- Remove $\{D, E, G\}$, which is consumed by $\{C, D, E\}$



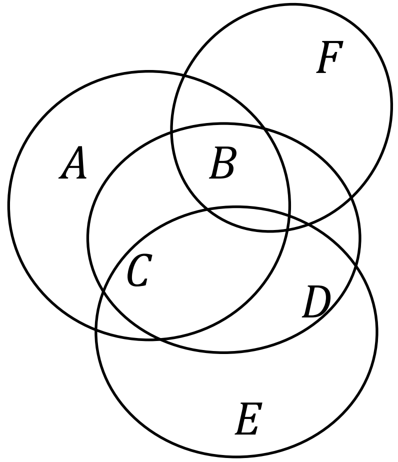
Join Tree 2



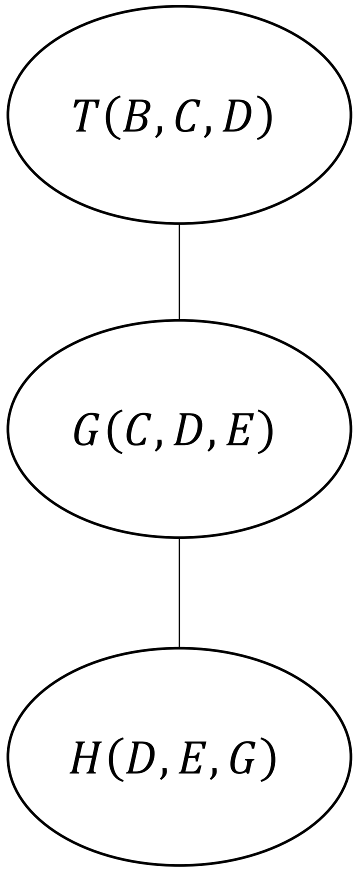
- Start to eliminate $\{D, E, G\}$
- Since $\{C, D, E\}$ consumes $\{D, E, G\}$, $\{C, D, E\}$ is the parent of $\{D, E, G\}$



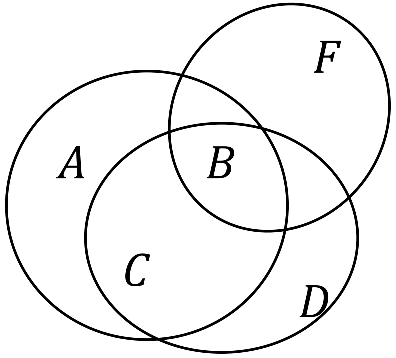
Join Tree 2



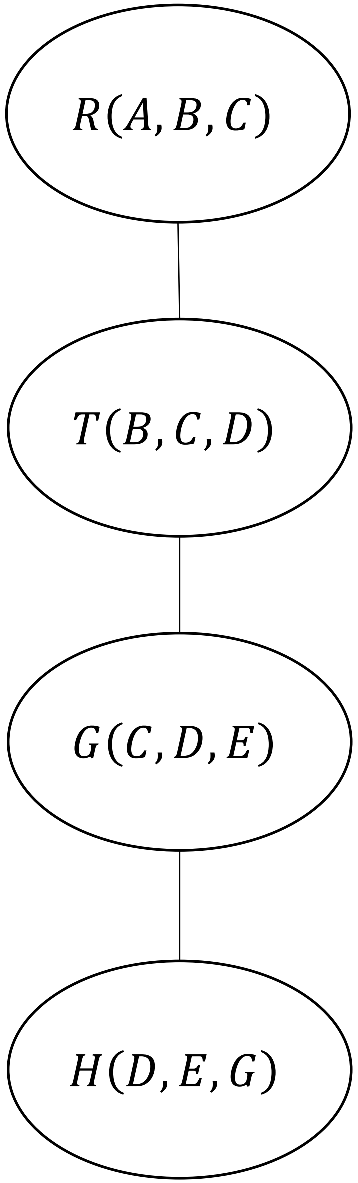
- Start to eliminate $\{D, E, G\}$
- Since $\{C, D, E\}$ consumes $\{D, E, G\}$, $\{C, D, E\}$ is the parent of $\{D, E, G\}$
- Remove $\{C, D, E\}$, which is consumed by $\{B, C, D\}$



Join Tree 2

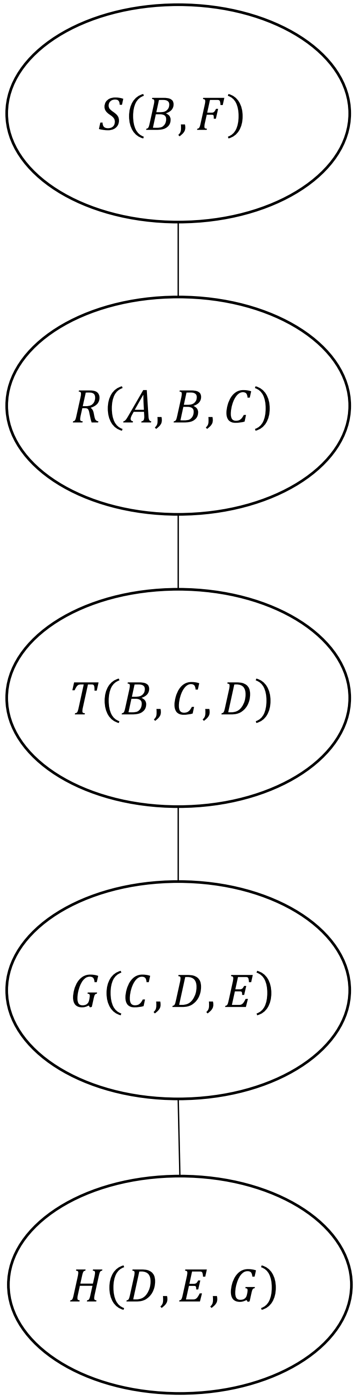


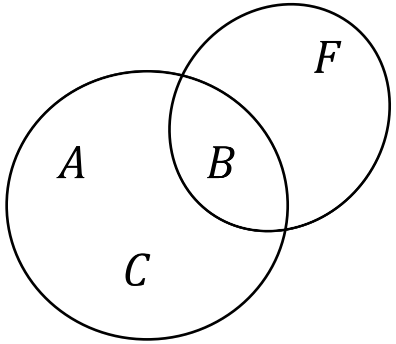
- Start to eliminate $\{D, E, G\}$
- Since $\{C, D, E\}$ consumes $\{D, E, G\}$, $\{C, D, E\}$ is the parent of $\{D, E, G\}$
- Remove $\{C, D, E\}$, which is consumed by $\{B, C, D\}$
- Remove $\{B, C, D\}$, which is consumed by $\{A, B, C\}$



Join Tree 2

- Start to eliminate $\{D, E, G\}$
- Since $\{C, D, E\}$ consumes $\{D, E, G\}$, $\{C, D, E\}$ is the parent of $\{D, E, G\}$
- Remove $\{C, D, E\}$, which is consumed by $\{B, C, D\}$
- Remove $\{B, C, D\}$, which is consumed by $\{A, B, C\}$
- Remove $\{A, B, C\}$ and $\{B, F\}$ sequentially





Complexity Notation

- Standard O and Ω notation for time and memory complexity in the RAM model of computation
- Use $\tilde{O}$-notation (soft-O)
  - Abstracts away polylog factors in input size that clutter formulas
  - $O\left(n^{f(l)} + (\log n)^{f(l)} \cdot r\right)$ becomes $\tilde{O}\left(n^{f(l)} + r\right)$

Data Complexity

- Complexity in query grows in two dimensions:
  - size of query (i.e., number of relations in a multi-way join query)
  - database size (i.e., number of rows contained in each relation of the query)
- Data complexity: the query is fixed (i.e., the size of the query expression itself l as a constant), and the complexity is expressed in terms of the size of database
- Suppose the query Q size $|Q|$ is l, then $O\left(f(l) \cdot n^{f(l)} + (\log n)^{f(l)} \cdot r\right)$ with $f()$ denote some arbitrary computable function can be simplified to $O\left(n^{f(l)} + (\log n)^{f(l)} \cdot r\right)$

Lower Bound for Any Join Algorithm

- Join output result size cardinality: r
- Query size l (i.e., number of relations in join query)
- $\Omega(n + r)$ data complexity to compute any query
  - The join algorithm has to read entire input at least once $\Omega(ln)$ (data complexity: $\Omega(n)$)
  - The join algorithm has to output result $\Omega(lr)$ (data complexity: $\Omega(r)$)
    - This the cost of concatenating tuples from l relations to form the final join result set
- Yannakakis algorithm amazingly matches the lower bound for acyclic CQs with data complexity $\tilde{O}(n + r)$

Yannakakis Algorithm

- Given acyclic conjunctive query represented by a join tree
- Two Phases
  - Apply a full reducer based on join tree
    - Semi-join reduction sweep from leaves to root
    - Semi-join reduction sweep from root to leaves
  - Use the join tree as the query plan and compute the joins bottom up

Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

1. Bottom-up traversal (semi-joins)
2. Top-down traversal (semi-joins)

R_1

A_1	A_2
1	20
1	10

R_2

A_1	A_2	A_3
1	10	100
1	20	100

R_3

A_2
10
20

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000

Slides of this example are from DATA Lab@Northeastern University

Example

$$Q = R_1(A_1, A_2) \bowtie R_2(A_1, A_2, A_3) \bowtie R_3(A_2) \bowtie R_4(A_1, A_2, A_4)$$

1. Bottom-up traversal (semi-joins)
2. Top-down traversal (semi-joins)
3. Join bottom-up

$$R_2 = R_3 \bowtie R_2$$

$$R_2 = R_4 \bowtie R_2$$

$$R_1 = R_1 \bowtie R_2$$

R_1

A_1	A_2
1	20
1	10

R_2

A_1	A_2	A_3
1	10	100
1	20	100

R_3

A_2
10
20

R_4

A_1	A_2	A_4
1	10	1000
1	20	1000
1	20	2000

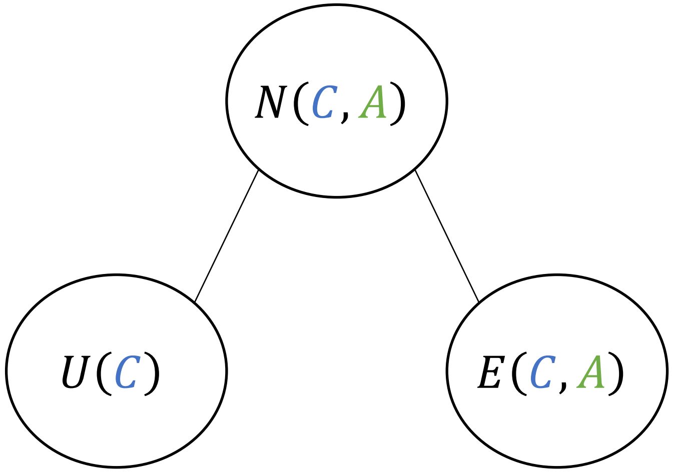
Slides of this example are from DATA Lab@Northeastern University

Yannakakis Algorithm Property

- Key Property
  - No intermediate join result size can be larger than the final result size
  - i.e., each join step can never shrink intermediate result size
- Why?
  - Semi-join reduction removes dangling tuples between pair-wise relations
  - Is it sufficient? No!
  - We need *connectedness condition* from join tree to ensure all dangling tuples are removed by semi-join reductions

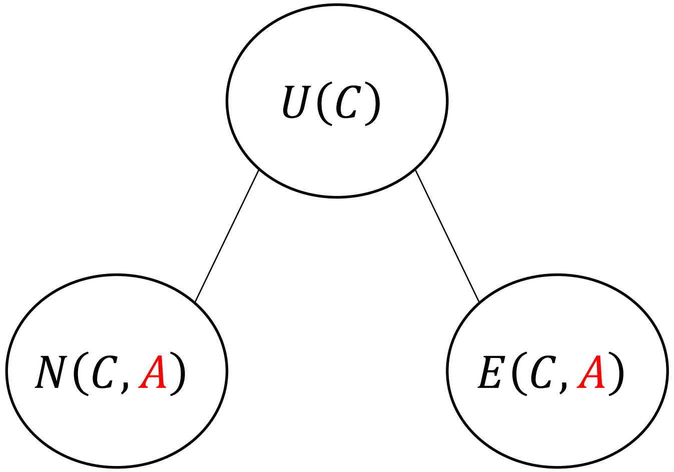
Importance of *connectedness condition*

- Suppose we have a database instance of $\{N(\text{“}Navy\text{”}, 13), U(\text{“}Navy\text{”}), E(\text{“}Navy\text{”}, 17)\}$
- Final join result: $\emptyset$



$\mathrm{N} \ltimes U, N \ltimes \mathrm{E}, \mathrm{U} \ltimes \mathrm{N}, \mathrm{E} \ltimes N$

$U = \emptyset, N = \emptyset, E = \emptyset$



$\mathrm{N} \ltimes U, E \ltimes \mathrm{U}, \mathrm{U} \ltimes \mathrm{N}, U \ltimes E$

$U = \{(\text{“}Navy\text{”})\}, N = \{(\text{“}Navy\text{”}, 13)\}, E = \{(\text{“}Navy\text{”}, 17)\}$

Yannakakis Algorithm Complexity

- Semi-join sweeps take $\tilde{O}(n)$
  - Recall $R \ltimes \mathrm{S} = \pi_{attr(R)}(R \bowtie S)$
  - With sort-merge join, we can compute $R \ltimes \mathrm{S}$ in $O(n \log n) = \tilde{O}(\mathrm{n})$
  - There are $2l - 2$ pair-wise semi-join operation, $\tilde{O}\big((2\mathrm{l} - 2)\mathrm{n}\big) = \tilde{O}(\mathrm{n})$ in data complexity
- All intermediate results are of size $O(r)$ b/c key property
- Each join step has $O(n + r)$ input and $O(r)$ output, which can be computed in $\tilde{O}(n + r)$ by sort-merge join (l join steps but ignored in data complexity)
- In total, Yannakakis Algorithm takes $\tilde{O}(n + r)$

Worst-Case Optimal Join Algorithm

Zeyuan Hu

May 3rd, 2021

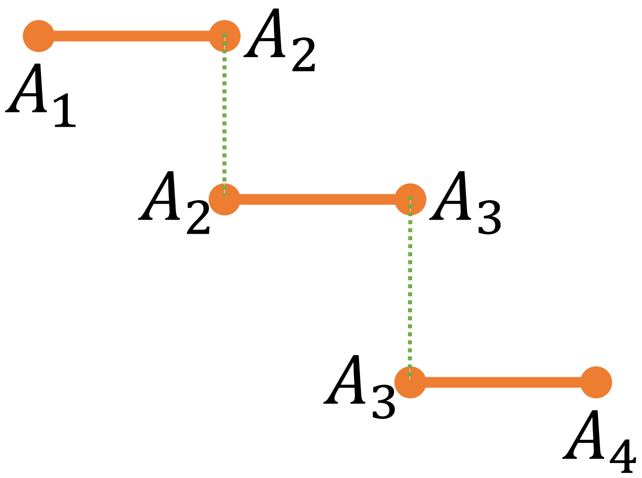
Recap

- Three properties for acyclic query
  1. It has a join tree, or
  2. It has a full reducer, or
  3. Its hypergraph is acyclic
- How to construct a full reducer from a join tree
- Modify GYO algorithm to construct join tree
- Yannakakis algorithm can run in $\tilde{O}(n + r)$ for acyclic CQ

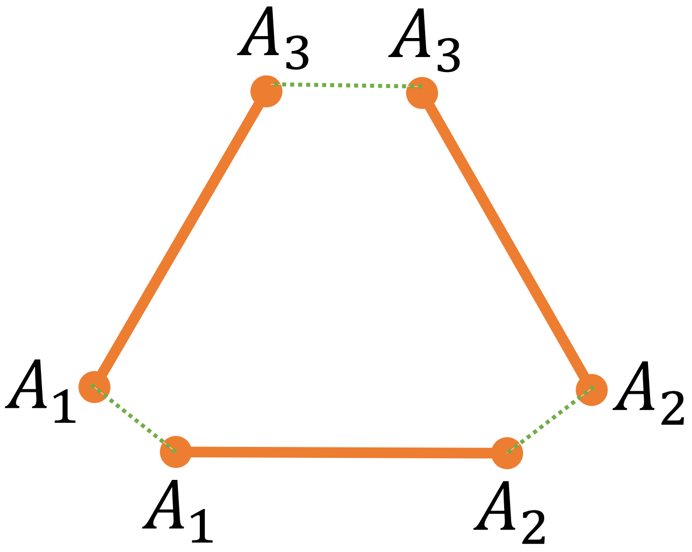
CQs with Cycles

- 3-path: $Q_{3p} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_4)$
- 3-cycle: $Q_{3c} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_1)$

Q_{3p}



Q_{3c}



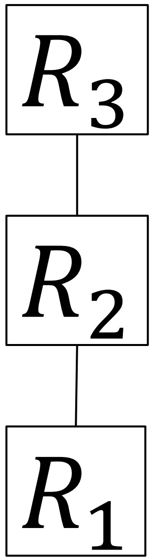
Slides of this example are from DATA Lab@Northeastern University

What's Wrong with Cyclic CQ

- Essentially, we cannot find an acyclic query graph that meets *connectedness condition*
  - → intermediate results size can be larger than the final result size
  - → key property of Yannakakis Algorithm falls through
- Example
  - 3-path: $Q_{3p} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_4)$
  - 3-cycle: $Q_{3c} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_1)$

What’s Wrong with Cyclic CQ (cont’)

Query Graph



- 3-path: $Q_{3p} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_4)$
- 3-cycle: $Q_{3c} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_1)$
- Already semi-join-reduced input

R_1

A_1	A_2
1	1
2	1
…	…
n	1

R_2

A_2	A_3
1	1
1	2
…	…
1	n

R_3

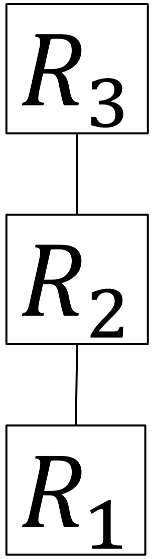
A_3	*
1	1
2	2
…	…
n	n

Slides of this example are from DATA Lab@Northeastern University

What's Wrong with Cyclic CQ (cont')

- 3-path: $Q_{3p} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_4)$
- 3-cycle: $Q_{3c} = R_1(A_1, A_2) \bowtie R_2(A_2, A_3) \bowtie R_3(A_3, A_1)$
- Already semi-join-reduced input
- $R_1 \bowtie R_2$ produces n^2 intermediate results
  - Final output size: n^2 for Q_{3p}, but only n for Q_{3c}

Query Graph



R_1

A_1	A_2
1	1
2	1
…	…
n	1

R_2

A_2	A_3
1	1
1	2
…	…
1	n

R_3

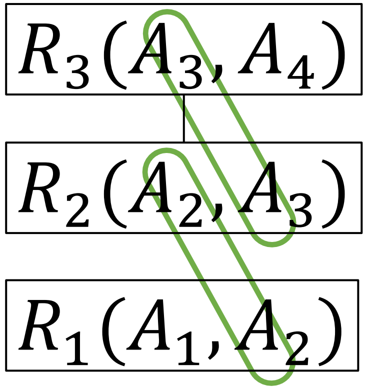
A_3	*
1	1
2	2
…	…
n	n

Slides of this example are from DATA Lab@Northeastern University

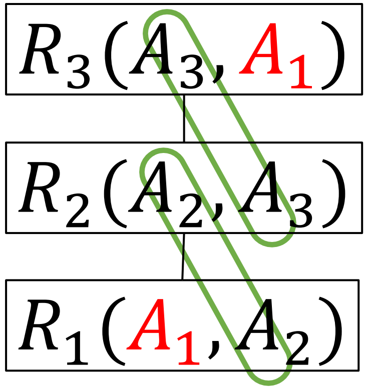
What's Wrong with Cyclic CQ (cont')

- Both queries have acyclic query graph
- In the right tree, A_1 violates *connectedness condition*

Q_{3p}

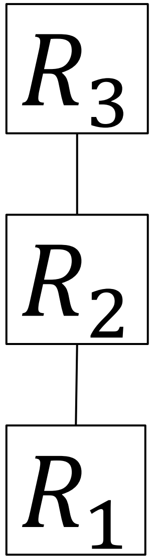


Q_{3c}



- Q_{3p} 's query graph is a join tree

Query Graph



Solutions for Cyclic CQ?

- Maybe we just need an algorithm that targets at Cyclic CQ?
- A result that is from '18 by Ngo et al shows that $\widetilde{\mathrm{O}}(n + r)$ is unattainable for full CQ based on well-accepted complexity-theoretic assumptions (e.g., P != NP)

What Can Be Done?

- Two main ideas
  - Worst-case Optimal Join Algorithms (WCOJA)
  - Tree decompositions
- Tree decompositions
  - Break down a cyclic CQ into query fragments called “bags”
  - Evaluate each query fragment using WCOJA and materialize the result
  - Connect bag results as a join tree and evaluate the whole query using Yannakakis algorithm
- We will focus on WCOJA

Theory of Computation Revisit

- Query evaluation problem is known to be NP-Complete
  - No algorithm exists to evaluate <u>any possible query</u> correctly and runs in polynomial time
  - Not a death sentence yet!
  - NP-Complete → algorithm cannot have <u>all</u> three properties
    - *General purpose.* The algorithm accommodates all possible inputs of the computational problem
    - *Correct.* For every input, the algorithm correctly solves the problem.
    - *Fast.* For every input, the algorithm runs in polynomial time.
- Choose one to compromise – General Purpose → Yannakakis Algorithm
- WCOJA chooses different to compromise - Fast

Query Evaluation Problem

- Given
  - a full CQ of the form $q = R_1(\overline{A_1}) \bowtie R_2(\overline{A_2}) \bowtie \ldots \bowtie R_m(\overline{A_m})$ where $\overline{A_j}$ is the attribute set of relation R_j, $j \in [m]$
  - a database instance I on the schema $\{R_1, \ldots, R_m\}$
- Query evaluation problem is to compute $q(I)$
  - $q(I)$ = a set of tuples $\boldsymbol{t}$ over attribute set $\bigcup_{j\in[m]} A_j$ s.t. projection of $\boldsymbol{t}$ onto the attributes $\overline{A_j}$ belongs to R_j, for each $j \in [m]$
- Join output result size cardinality: r
  - r is database instance dependent
- Yannakakis Algorithm reaches $\tilde{O}(n + r)$

Optimal Worst-case Join Evaluation Problem

- An easier problem than query evaluation problem
- Instead of $\tilde{O}(n+r)$, hope to find a polynomial algorithm that can run $\tilde{O}(n+r_{WC})$
  - r_{WC} = maximum possibly output size for the given size of the relations in q
- Let $\bar{N} = \{N_1, \ldots, N_m\}$ and let $I(\bar{N})$ be the set of database instances with $|R_j^I| = N_j$ for $j \in [m]$. Then, $r_{WC} = \sup_{I \in I(\bar{N})} |q(I)|$
  - i.e., supremum (maximum) of all possible r over $I(\bar{N})$
- Even database instance has the same size, the distribution of data can be different and thus we can get different join output size

AGM Bound

- Example:
  - $Q(a,b,c) \leftarrow R(a,b), S(b,c), T(a,c)$
- How large is r_{WC}?
  - Given the sizes of $|R|, |S|, \text{and } |T|$, what is the largest possible query result size r?
- Solved by Aterias, Grohe, and Marx in ‘08
- We’ll introduce intuition here

AGM Bound Intuition

- Given $Q(a, b, c) \leftarrow R(a, b), S(b, c), T(a, c)$ and $|R| = |S| = \ |T| = N$, what is the bound on the query result size?
- One bound is $O(N^3)$ because we have three-way join and each tuple can be part of final join result. Thus, we have a cartesian product.
- Can we do better? Yes! $O(N^2)$
- Observe that join of any two relations is an upper bound on r
  - Because we have a triangle query, third relation imposes additional constraint on intermediate relation, which can at best not eliminate any tuples from intermediate relation.
  - $R(a, b) \ \bowtie S(b, c)$ already gives tuples with attributes (a, b, c), introduce T can remove tuples

AGM Bound Intuition (cont')

- For $Q(a,b,c) \leftarrow R(a,b), S(b,c), T(a,c)$, AGM bound gives $O(N^{1.5})$
- How? By generalizing the observation we have for Q using *fractional edge cover*
- Edge cover: a set of edges s.t. each vertex in graph G is an end of at least one edge
- AGM formulate a linear programming problem based on edge cover of hypergraph of Q. Solving such problem leads to the bound.

WCOJA (under graph model)

- We’ll describe WCOJA in the context of graph model using graph pattern matching query (i.e., subgraph query)
- A *match* is a mapping from variables to constants such that when the mapping is applied to the given pattern, the result is, roughly speaking, contained within the original graph (i.e., subgraph).
- Focus on triangle query
  - $Q(a,b,c) \leftarrow R(a,b), S(b,c), T(a,c)$
  - In Cypher syntax
    - `match (a)-[:TO]->(b)-[:TO]->(c)-[:TO]->(a) return distinct a, b, c`

Edges	
Vi	Vj
A	B
D	B
B	C

Relational View of Subgraph Queries

- We have seen in Cypher that subgraph query = multi-way join query
- Suppose we use $Edges$ relation to store the input graph G
  - $Edges$ relation contains every directed edges in G
- Query to find all directed triangles in G
  - $Q(a_1, a_2, a_3) \leftarrow Edges(a_1, a_2), Edges(a_2, a_3), Edges(a_3, a_1)$

Evaluate Triangle Query: Traditional Approach

- Traditional Approach
  - Treat subgraph query as relational query
  - Evaluate the query using a sequence of binary joins
  - “Edge-at-a-time” approach
- We have seen because of break of *connectedness condition*, intermediate results can be greater than final result
- From acyclicity, you might sense some connection between query representation and query processing algorithm
  - Join tree (loosely, query graph) → pair-wise binary joins (Yannakakis)
  - Hypergraph → vertex-at-a-time approach

Generic Join (GJ) as a WCOJA

GJ consists of the following three high-level ingredients

- Global Attribute Ordering
  - GJ first orders the attributes. For example, we assume the orders $a_1, \ldots, a_m$
- Extension Indices
  - *Prefix j-tuple* = any fixed values of the first $j < m$ attributes
    - For each R_i and j-tuple p only some values for attribute a_{j+1} exist in R_i
  - *Extension index* Ext_j^i map each j-tuple p to values of a_{j+1} matching p in R_i
    - $Ext_j^i\colon \left(p = \left(a_1, \ldots, a_j\right)\right) \to \{a_{j+1}\}$
    - Each relation has its own extension index
    - Such index needs to have some certain properties to enable GJ reaching $\tilde{O}(n + r_{WC})$

Generic Join (GJ) as a WCOJA (cont')

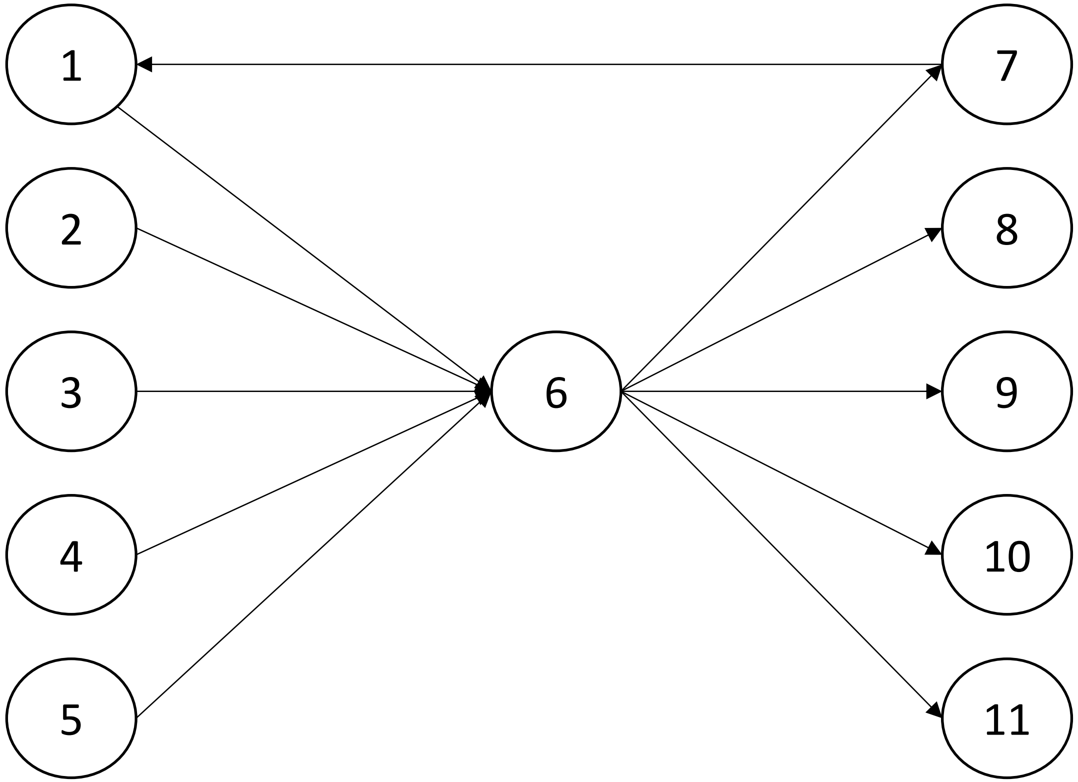
- Prefix Extension Stages
  - GJ iteratively computes intermediate results $P_1, \ldots, P_m$
    - P_j = result of Q when each relation is restricted to the first j attributes in the global order
  - GJ starts from the singleton relation P_0 with no attributes
  - P_m is the final join result for Q
  - GJ determines P_{j+1} from P_j using the extension indices
    - For each j-tuple $p \in P_j$, GJ first intersects Ext_j^i of each relation R_i containing a_{j+1}
    - The result of intersection is added to P_{j+1}
    - Intersection is performed from the smallest Ext_j^i to ensure algorithm runtime bound

Generic Join (GJ) Pseudocode

$P_0 = \{\}$
for $(j = 1 \ldots m)$:
    $P_j = \{\}$
    for $(p \in P_{j-1})$:
        // $\cap$ below is performed starting from smallest $Ext_j^i(p)$
        $ext_p = \cap Ext_j^i(p)$
        $P_j = P_j \cup ext_p$

Example

- $Q(a_1, a_2, a_3) \leftarrow R_1(a_1, a_2), R_2(a_2, a_3), R_3(a_3, a_1)$
- R_1, R_2, R_3 are all $Edges$ relation

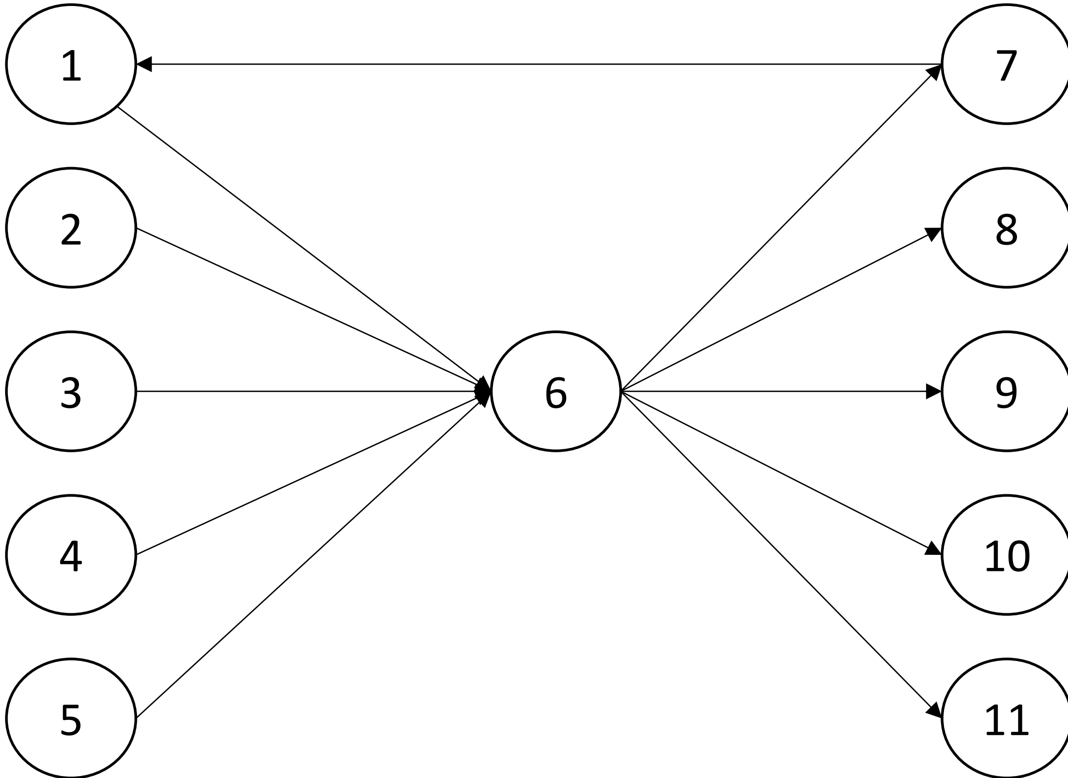


Example

- The global attribute ordering is a_1, a_2, a_3
- GJ starts with $P_0 = \{\varepsilon\}$
- GJ next computes P_1
    - There is only one tuple in P_0, which is empty
    - Only R_1 and R_3 contain a_1
        - $Ext_0^1 = \{1,2,3,4,5,6,7\}$
        - $Ext_0^3 = \{1,6,7,8,9,10,11\}$
        - $Ext_0^1 \cap Ext_0^3 = \{1,6,7\}$
    - $\varepsilon \times \{1,6,7\} = \{(1),(6),(7)\}$
    - $P_1 = \{\quad\} \cup \{(1),(6),(7)\} = \{(1),(6),(7)\}$
    - No more tuple left in P_0, done with P_1

```
P0={}
for (j = 1... m):
    Pj={}
    for (p ∈ Pj−1):
        // ∩ below is performed starting from smallest Ext^i_j(p)
        ext_p = ∩Ext^i_j(p)
        Pj = Pj ∪ ext_p
```

$$Q(a_1, a_2, a_3) \leftarrow R_1(a_1, a_2), R_2(a_2, a_3), R_3(a_3, a_1)$$

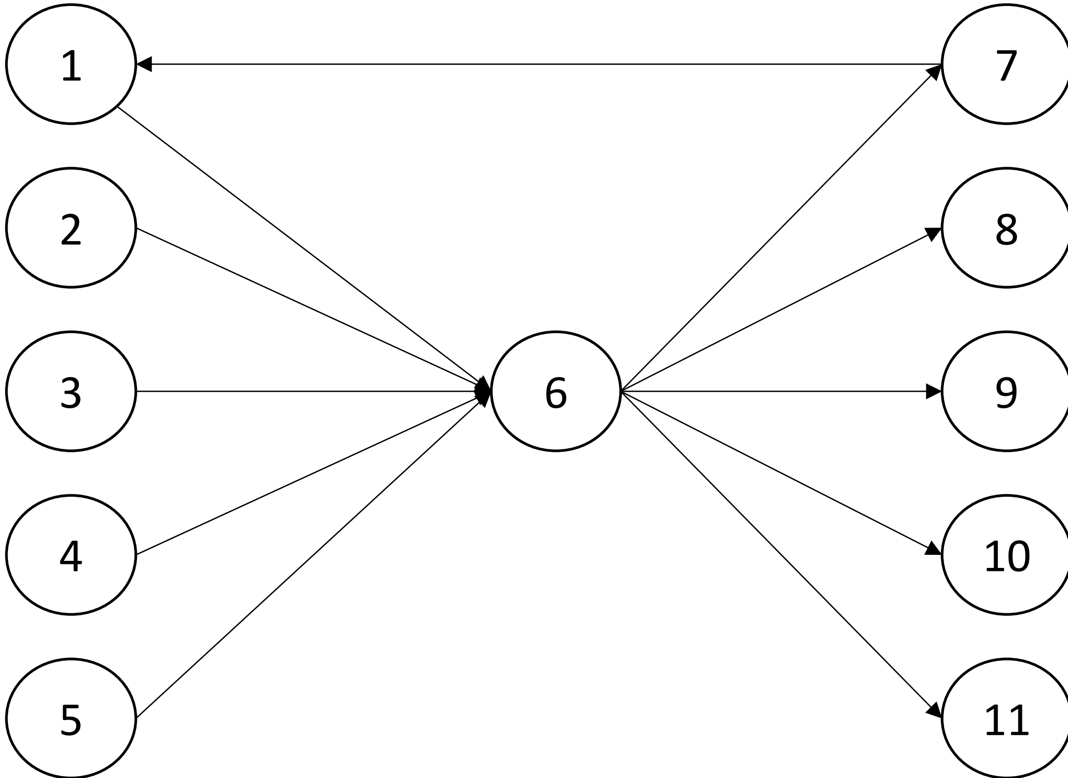


Example

- $P_1 = \{(1), (6), (7)\}$
- GJ next computes P_2
- R_1 and R_2 contain a_2
- Start with (1)
    - $Ext_1^1 = \{6\}$
    - $Ext_1^2 = \{1,2,3,4,5,6,7\}$
    - $Ext_1^1 \cap Ext_1^2 = \{6\}$
  - (1) $\times \{6\} = \{(1,6)\}$
  - $P_2 = \{\quad\} \cup \{(1,6)\} = \{(1,6)\}$
  - More tuple left in P_1, continue

```
P_0={}
for (j = 1... m):
    P_j={}
    for (p ∈ P_{j-1}):
        // ∩ below is performed starting from smallest Ext_j^i(p)
        ext_p = ∩Ext_j^i(p)
        P_j = P_j ∪ ext_p
```

$Q(a_1, a_2, a_3) \leftarrow R_1(a_1, a_2), R_2(a_2, a_3), R_3(a_3, a_1)$

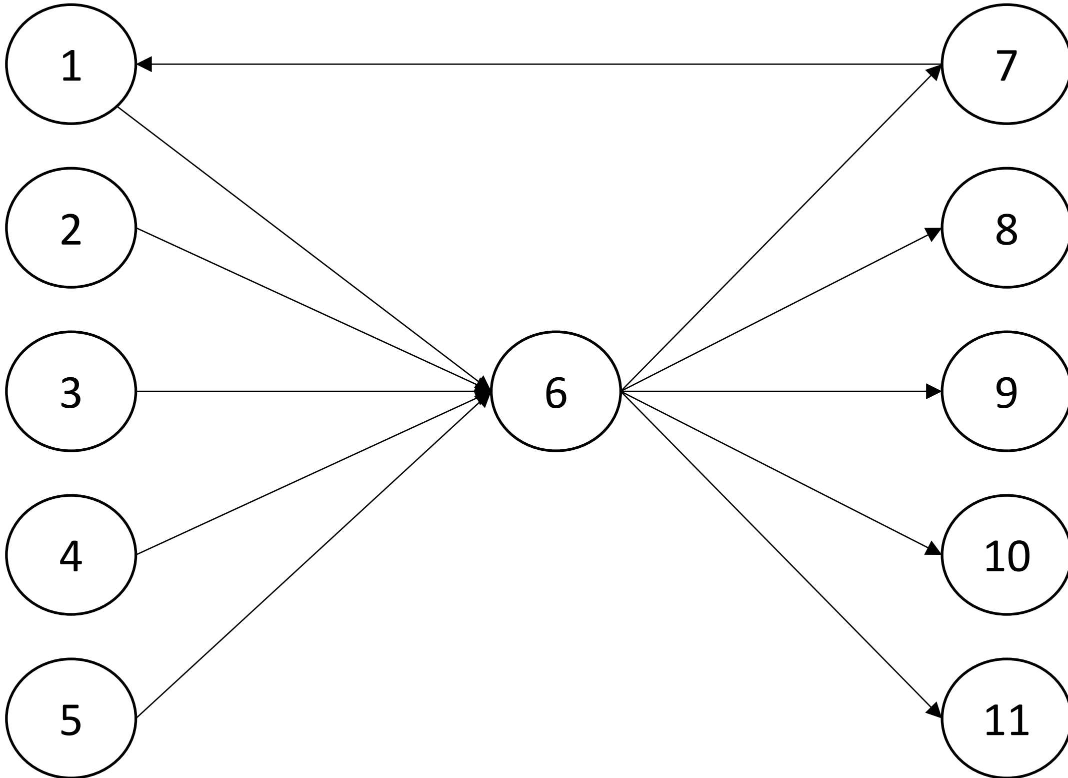


Example

- $P_1 = \{(1), (6), (7)\}$
- GJ next computes P_2
- R_1 and R_2 contain a_2
- Next, (6)
    - $Ext_1^1 = \{7,8,9,10,11\}$
    - $Ext_1^2 = \{1,2,3,4,5,6,7\}$
    - $Ext_1^1 \cap Ext_1^2 = \{7\}$
  - $(6) \times \{7\} = \{(6,7)\}$
  - $P_2 = \{(1,6)\} \cup \{(6,7)\} = \{(1,6), (6,7)\}$
  - More tuple left in P_1, continue

```
P_0={}
for (j = 1... m):
    P_j={}
    for (p ∈ P_{j−1}):
        // ∩ below is performed starting from smallest Ext_j^i(p)
        ext_p = ∩Ext_j^i(p)
        P_j = P_j ∪ ext_p
```

$Q(a_1, a_2, a_3) \leftarrow R_1(a_1, a_2), R_2(a_2, a_3), R_3(a_3, a_1)$

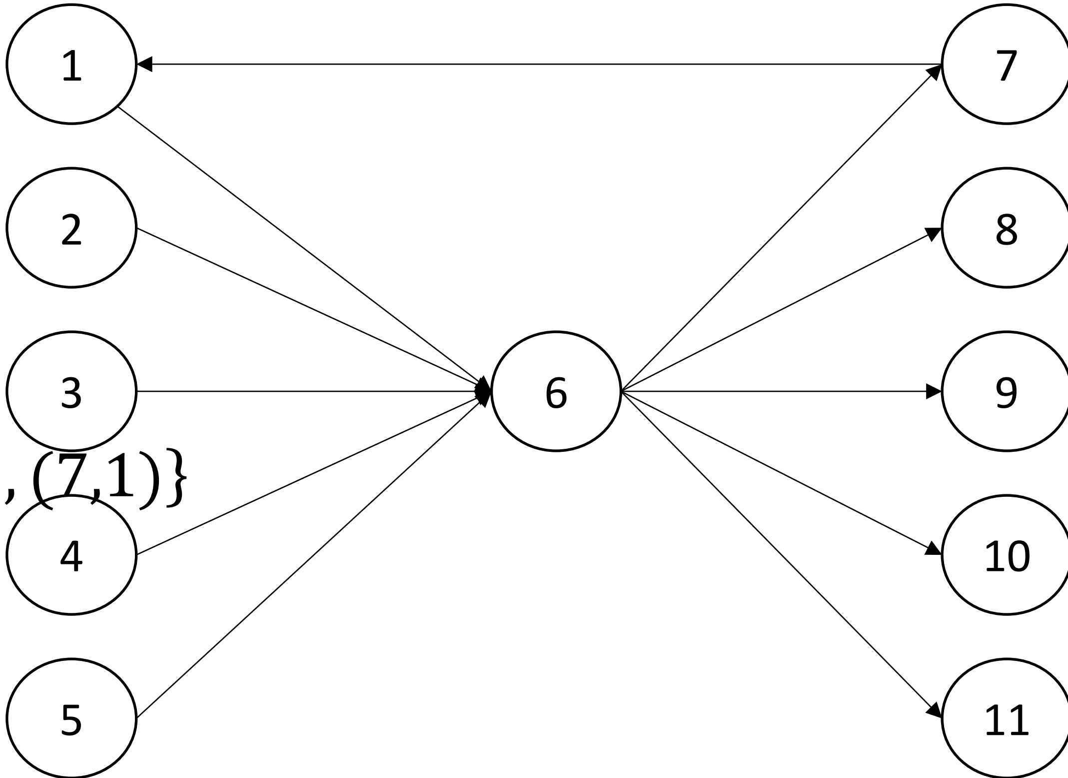


Example

- $P_1 = \{(1), (6), (7)\}$
- GJ next computes P_2
- R_1 and R_2 contain a_2
- Next, (7)
        - $Ext_1^1 = \{1\}$
        - $Ext_1^2 = \{1,2,3,4,5,6,7\}$
        - $Ext_1^1 \cap Ext_1^2 = \{1\}$
    - $(7) \times \{1\} = \{(7,1)\}$
    - $P_2 = \{(1,6),(6,7)\} \cup \{(7,1)\} = \{(1,6),(6,7),(7,1)\}$
    - No more tuple left in P_1, done with P_2

```
P0={}
for (j = 1... m):
    Pj={}
    for (p ∈ Pj−1):
        // ∩ below is performed starting from smallest Ext^i_j(p)
        ext_p = ∩Ext^i_j(p)
        Pj = Pj ∪ ext_p
```

$Q(a_1, a_2, a_3) \leftarrow R_1(a_1, a_2), R_2(a_2, a_3), R_3(a_3, a_1)$

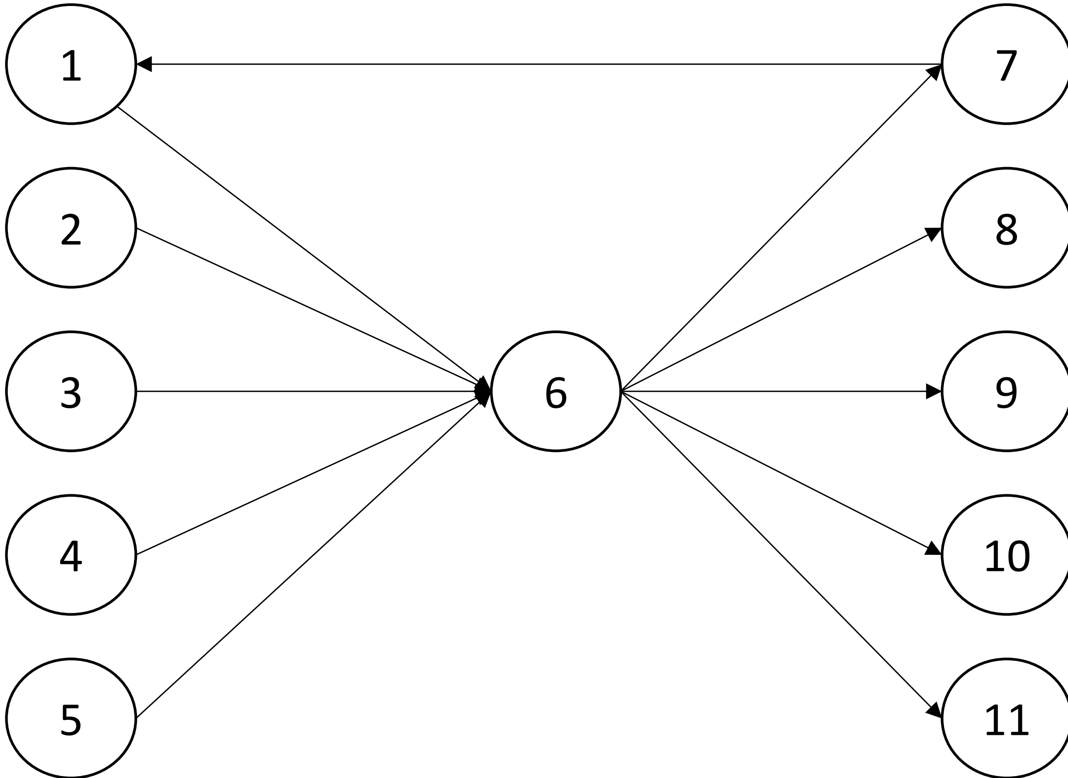


Example

- $P_2 = \{(1,6),(6,7),(7,1)\}$
- GJ next computes P_3
- R_2 and R_3 contain a_3
- First, $(1,6)$
    - $Ext_2^2 = \{7,8,9,10,11\}$
    - $Ext_2^3 = \{7\}$
    - $Ext_2^2 \cap Ext_2^3 = \{7\}$
  - (7) $\times\{(1,6)\} = \{(1,6,7)\}$
  - $P_3 = \{\quad\} \cup \{(1,6,7)\} = \{(1,6,7)\}$
  - More tuple left in P_2, continue

```
P0={}
for (j = 1... m):
    Pj={}
    for (p ∈ Pj−1):
        // ∩ below is performed starting from smallest Ext^i_j(p)
        ext_p = ∩Ext^i_j(p)
        Pj = Pj ∪ ext_p
```

$Q(a_1,a_2,a_3) \leftarrow R_1(a_1,a_2), R_2(a_2,a_3), R_3(a_3,a_1)$

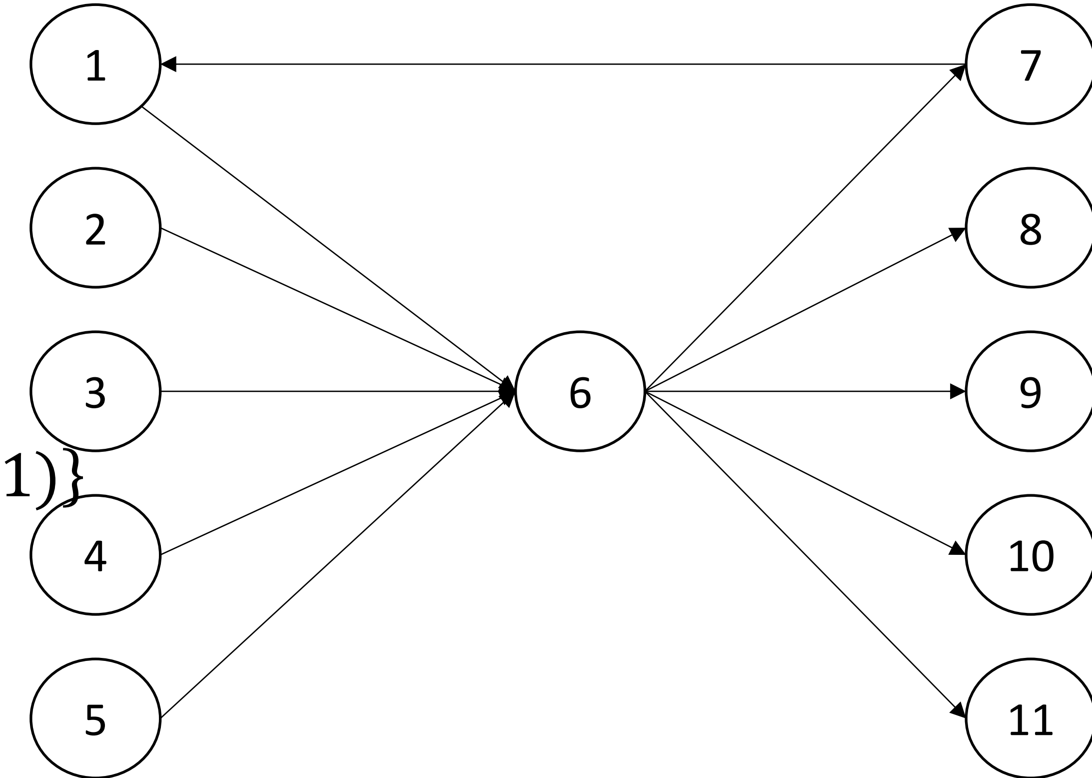


Example

- $P_2 = \{(1,6), (6,7), (7,1)\}$
- GJ next computes P_3
- R_2 and R_3 contain a_3
- Next, $(6,7)$
    - $Ext_2^2 = \{1\}$
    - $Ext_2^3 = \{1,2,3,4,5\}$
    - $Ext_2^2 \cap Ext_2^3 = \{1\}$
  - $(1) \times \{(6,7)\} = \{(6,7,1)\}$
  - $P_3 = \{(1,6,7)\} \cup \{(6,7,1)\} = \{(1,6,7), (6,7,1)\}$
  - More tuple left in P_2, continue

P_0={}
for ($j = 1 \ldots m$):
  P_j={}
  for ($p \in P_{j-1}$):
    // $\cap$ below is performed starting from smallest $Ext_j^i(p)$
    $ext_p = \cap Ext_j^i(p)$
    $P_j = P_j \cup ext_p$

$Q(a_1, a_2, a_3) \leftarrow R_1(a_1, a_2), R_2(a_2, a_3), R_3(a_3, a_1)$

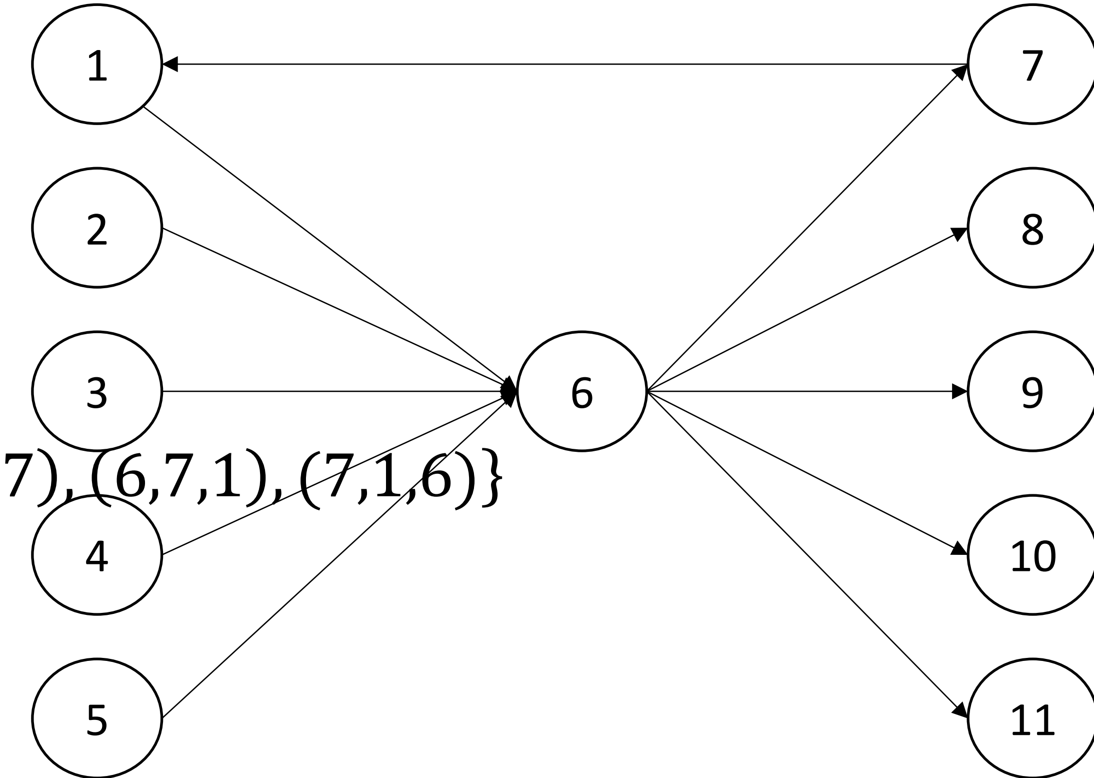


Example

- $P_2 = \{(1,6),(6,7),(7,1)\}$
- GJ next computes P_3
- R_2 and R_3 contain a_3
- Next, $(7,1)$
    - $Ext_2^2 = \{6\}$
    - $Ext_2^3 = \{6\}$
    - $Ext_2^2 \cap Ext_2^3 = \{6\}$
  - $(6) \times \{(7,1)\} = \{(7,1,6)\}$
  - $P_3 = \{(1,6,7),(6,7,1)\} \cup \{(7,1,6)\} = \{(1,6,7),(6,7,1),(7,1,6)\}$
  - No more tuple left in P_2, done with P_3

```
P_0={}
for (j = 1... m):
    P_j={}
    for (p ∈ P_{j−1}):
        // ∩ below is performed starting from smallest Ext_j^i(p)
        ext_p = ∩Ext_j^i(p)
        P_j = P_j ∪ ext_p
```

$Q(a_1,a_2,a_3) \leftarrow R_1(a_1,a_2), R_2(a_2,a_3), R_3(a_3,a_1)$



Final Remarks

- In our example, since each attribute in the ordering is contained in two relations, $\cap Ext_j^i$ from the smallest doesn't apply but be aware
- Interested in time complexity proof (non-trivial), see "Skew strikes back: New developments in the theory of join algorithms" by Ngo et.al in 2014